Ableitungsregeln

- Faktorregel
- Summenregel
- Produktregel
- Quotientenregel
- Kettenregel

3 Differential- und Integralrechnung
3.1 Ableitungsregeln
3.1.1 Faktor- und Summenregel

Definition

$$\frac{d}{dx} f(x) = \frac{d}{dx}\left(a_1 f_1(x) + a_2 f_2(x)\right) = \frac{d}{dx} a_1 f_1(x) + \frac{d}{dx} a_2 f_2(x) = a_1 \frac{d}{dx} f_1(x) + a_2 \frac{d}{dx} f_2(x)$$

Beispiele

$\frac{d}{dx} x^n = n x^{n-1}$

$$f(x) = 2x^2 + \frac{2}{3}x^3$$

Summe

Faktor

$$\frac{d}{dx} f(x) = \frac{d}{dx}\left(2x^2 + \frac{2}{3}x^3\right) = \frac{d}{dx} 2x^2 + \frac{d}{dx}\frac{2}{3}x^3 = 2\frac{d}{dx}x^2 + \frac{2}{3}\frac{d}{dx}x^3 = 4x + 2x^2$$

$$f(x) = \frac{1}{2}x^2 + 2\sin(x)$$

$\frac{d}{dx}\sin(x) = \cos(x)$

$$\frac{d}{dx} f(x) = \frac{d}{dx}\left(\frac{1}{2}x^2 + 2\sin(x)\right) = \frac{d}{dx}\frac{1}{2}x^2 + \frac{d}{dx}2\sin(x) = \frac{1}{2}\frac{d}{dx}x^2 + 2\frac{d}{dx}\sin(x)$$

$$= x + 2\cos(x)$$

3 Differential- und Integralrechnung
3.1 Ableitungsregeln
3.1.2 Produktregel

Definition

$$\frac{d}{dx} f(x) = \frac{d}{dx}\big(u(x)v(x)\big) = v(x)\frac{d}{dx}u(x) + u(x)\frac{d}{dx}v(x) = u'(x)v(x) + u(x)v'(x)$$

Beispiele

$\frac{d}{dx}\sin(x) = \cos(x)$

$\frac{d}{dx}\cos(x) = -\sin(x)$

$$f(x) = \sin(x)\cos(x)$$

$$\frac{d}{dx} f(x) = \frac{d}{dx}\big(\sin(x)\cos(x)\big) = \cos(x)\underbrace{\frac{d}{dx}\sin(x)}_{u'(x)} + \sin(x)\underbrace{\frac{d}{dx}\cos(x)}_{v'(x)}$$

$$= \cos^2(x) - \sin^2(x)$$

$\frac{d}{dx}e^x = e^x$

$$f(x) = e^x \sin(x)$$

$$\frac{d}{dx} f(x) = \frac{d}{dx}\big(e^x \sin(x)\big) = \sin(x)\underbrace{\frac{d}{dx}e^x}_{u'(x)} + e^x\underbrace{\frac{d}{dx}\sin(x)}_{v'(x)}$$

$$= e^x \sin(x) + e^x \cos(x) = e^x\big(\sin(x) + \cos(x)\big)$$

$$f(x) = e^x \sin(x)\cos(x)$$

$$\frac{d}{dx} f(x) = \frac{d}{dx}\big(e^x \sin(x)\cos(x)\big) = \underbrace{\big(e^x \sin(x)\big)}_{u(x)}\underbrace{\big(\cos(x)\big)}_{v(x)} = \underbrace{\big(e^x\big)}_{u(x)}\underbrace{\big(\sin(x)\cos(x)\big)}_{v(x)}$$

3 Differential- und Integralrechnung
3.1 Ableitungsregeln
3.1.3 Produktregel

Definition

$$\frac{d}{dx} f(x) = \frac{d}{dx}\big(u(x)v(x)\big) = v(x)\frac{d}{dx}u(x) + u(x)\frac{d}{dx}v(x) = u'(x)v(x) + u(x)v'(x)$$

Beispiele

$f(x) = e^x \sin(x)\cos(x)$

$u(x)$ $v(x)$ $u(x)$ $v(x)$

$$\frac{d}{dx} f(x) = \frac{d}{dx}\big(e^x \sin(x)\cos(x)\big) = \big(e^x \sin(x)\big)\big(\cos(x)\big) = \big(e^x\big)\big(\sin(x)\cos(x)\big)$$

$$= \big(e^x \sin(x)\big)'\big(\cos(x)\big) + \big(e^x \sin(x)\big)\big(\cos(x)\big)'$$

$$= e^x\big(\sin(x) + \cos(x)\big)\cos(x) - e^x \sin(x)\sin(x)$$

$$= e^x\big(\sin(x)\cos(x) + \cos^2(x) - \sin^2(x)\big)$$

3 Differential- und Integralrechnung
3.1 Ableitungsregeln
3.1.4 Quotientenregel

Definition

$$\frac{d}{dx} f(x) = \frac{d}{dx}\left(\frac{u(x)}{v(x)}\right) = \frac{v(x)\frac{d}{dx}u(x) - u(x)\frac{d}{dx}v(x)}{v^2(x)} = \frac{u'(x)v(x) - u(x)v'(x)}{v^2(x)}$$

Beispiele

$$f(x) = \tan(x) = \frac{\sin(x)}{\cos(x)}$$

$\frac{d}{dx}\sin(x) = \cos(x)$

$\frac{d}{dx}\cos(x) = -\sin(x)$

$$\frac{d}{dx} f(x) = \frac{\overbrace{\cos(x)}^{v(x)}\frac{d}{dx}\overbrace{\sin(x)}^{u(x)} - \overbrace{\sin(x)}^{u(x)}\frac{d}{dx}\overbrace{\cos(x)}^{v(x)}}{\cos^2(x)} = \frac{\cos(x)\cos(x) + \sin(x)\sin(x)}{\cos^2(x)}$$

$$= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)}$$

3 Differential- und Integralrechnung
3.1 Ableitungsregeln
3.1.5 Kettenregel

Definition

$$\frac{d}{dx} f(x) = \frac{d}{dx} f(g(x)) = \frac{d}{dg} f(g) \frac{d}{dx} g(x) = f'(g)g'(x)$$

Beispiele

$\frac{d}{dx}\sin(x) = \cos(x)$
$\frac{d}{dx}\cos(x) = -\sin(x)$

$$f(x) = e^{\sin(x)} \qquad g(x) = \sin(x)$$

$$\frac{d}{dx} f(x) = \frac{d}{dx} e^{\sin(x)} = \frac{d}{dx}\sin(x)\frac{d}{dg} e^{g} = \cos(x)e^{g} = \cos(x)e^{\sin(x)}$$

$\frac{d}{dx} e^{x} = e^{x}$

$$f(x) = a^{x} = e^{x\ln(a)}$$

$$\frac{d}{dx} f(x) = \frac{d}{dx} e^{x\ln(a)} = \frac{d}{dx} x\ln(a) \frac{d}{dg} e^{g} = \ln(a)\frac{d}{dx} x \frac{d}{dg} e^{g} = \ln(a)e^{x\ln(a)} = a^{x}\ln(a)$$

$$f(x) = x^{a} = e^{a\ln(x)}$$

$\frac{d}{dx}\ln(x) = \frac{1}{x}$

$$\frac{d}{dx} f(x) = \frac{d}{dx} e^{a\ln(x)} = \frac{d}{dx} a\ln(x) \frac{d}{dg} e^{g} = a\frac{d}{dx}\ln(x)\frac{d}{dg} e^{g}$$

$$= \frac{a}{x} e^{g} = \frac{a}{x} e^{a\ln(x)} = \frac{a}{x} x^{a} = ax^{a-1}$$

Beispiele

$$f(x)=\frac{u(x)w(x)}{v(x)}$$

$$f'(x)=\left(\frac{u(x)w(x)}{v(x)}\right)'=\frac{\big(u(x)w(x)\big)'v(x)-u(x)w(x)v'(x)}{v^2(x)}$$

$$=\frac{\big(u'(x)w(x)+u(x)w'(x)\big)v(x)-u(x)w(x)v'(x)}{v^2(x)}$$

$$=\frac{u'(x)w(x)v(x)+u(x)w'(x)v(x)-u(x)w(x)v'(x)}{v^2(x)}$$

$$f(x)=\sin(u(x)v(x))$$

$$f'(x)=\big(\sin(u(x)v(x))\big)'=\big(u(x)v(x)\big)'\cos(u(x)v(x))$$

$$=\big(u'(x)v(x)+u(x)v'(x)\big)\cos(u(x)v(x))$$

$$f(x)=e^{a\cdot u(x)+b\cdot v(x)}$$

$$f'(x)=\left(e^{a\cdot u(x)+b\cdot v(x)}\right)'=\big(a\cdot u(x)+b\cdot v(x)\big)'e^{a\cdot u(x)+b\cdot v(x)}$$

$$=\big(a\cdot u'(x)+b\cdot v'(x)\big)e^{a\cdot u(x)+b\cdot v(x)}$$